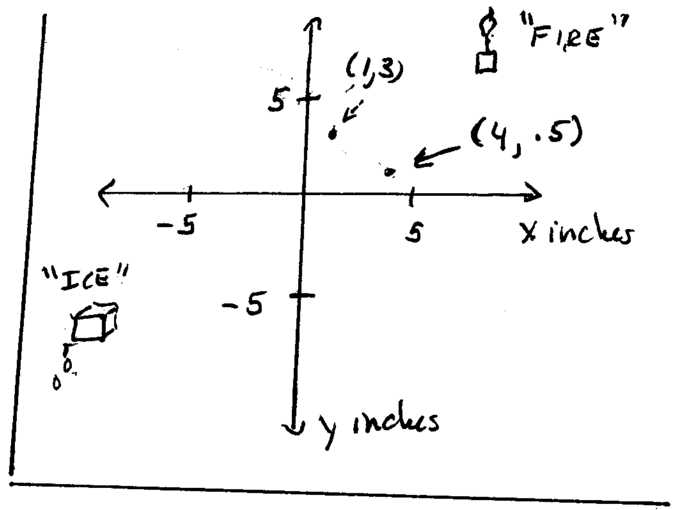
A Temperature Function and Its Partial Derivatives



Sources of heat are placed in QUADRANT I and sources of COLD ARE PLACED in QUADRANT III in such a manner that,

at every point (x,y) with $|x| < 5$ inches and $|y| < 5$ inches,

the Temperature $T(x,y)$ at that point is given by the function $T(x,y) = 70 + 3x + x^3 + 2y^3$ Degrees Farenheit ($^\circ$).

The partial derivatives of T are:

$$\frac{\partial T}{\partial x} = T_x(x,y) = 3 + 3x^2 \text{ Degrees per inch}$$

$$\frac{\partial T}{\partial y} = T_y(x,y) = 6y^2 \text{ Degrees per inch}$$

The calculations on page 2 will illustrate the fact that, at a point (x_0, y_0), the value of $T_x(x_0, y_0)$ is the rate at which the temperature T will change if the temperature T is calculated at a nearby point (x, y_0), holding y constant at $y = y_0$; i.e., $T_x(x_0, y_0)$ is the rate of change of T w.r.t. x at (x_0, y_0).

The calculations also illustrate the fact that, at (x_0, y_0), $T_y(x_0, y_0)$ is the rate at which T will change if T is calculated at a nearby point (x_0, y), holding x constant at $x = x_0$, i.e., $T_y(x_0, y_0)$ is the rate of change of T w.r.t. y at (x_0, y_0).

Recall: $T(x,y) = 70 + 3x + x^3 + 2y^3$

$T_x(x,y) = 3 + 3x^2$

$T_y(x,y) = 6y^2$

AT $(x_0, y_0) = (1,3)$, $T(1,3) = 128°$

$T_x(1,3) = 6°/\text{inch}$

$T_y(1,3) = 54°/\text{inch}$

	x	y	T(x,y)		
$\Delta x = .01$	1.00	3	128.000000	$\Delta T = .060301$	y is held constant
	1.01	3	128.060301	$\Delta T \approx 6 \times .01$	at y = 3
$\Delta y = .01$	1	3.00	128.000000	$\Delta T = .541802$	x is held constant
	1	3.01	128.541802	$\Delta T \approx 54 \times .01$	at x = 1

AT $(x_0, y_0) = (4, \frac{1}{2}) = (4, .5)$,

$T(4, .5) = 146.25°$

$T_x(4, .5) = 51°/\text{inch}$

$T_y(4, .5) = \frac{6}{4} = \frac{3}{2} = 1.5°/\text{inch}$

	x	y	T(x,y)		
$\Delta x = .01$	4.00	.5	146.250000	$\Delta T = .511201$	y is held constant at
	4.01	.5	146.761201	$\Delta T \approx 51 \times .01$	y = .5
$\Delta y = .01$	4	.50	146.250000	$\Delta T = .015302$	x is held constant at
	4	.51	146.265302	$\Delta T \approx 1.5 \times .01 = .015$	x = 4.